Year 8, 9 and 10 students' understanding and access of percent knowledge

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This paper reports on Years 8, 9 and 10 students' knowledge of percent problem types, type of solution strategy, and use of diagrams. Non- and semi-proficient students displayed the expected inflexible formula approach to solution but proficient students used a flexible mixture of estimation, number sense and trial and error instead of expected schema-based classification methods.

The application of percent in the real world cannot be denied. Percent discounts, profits, losses, savings, and increases, are an integral part of our society, as attested to by billboards, newspapers, advertisements, and retail marketing. There can be no question of the social necessity of having an understanding of percent and therefore its importance in the mathematics curriculum. However, percent is often misused or misunderstood when applied in the real world, as seen through errors made in media advertising (Watson, 1994).

According to Parker and Leinhardt (1995), one reason why percent is a difficult topic to learn and teach is that the notion of percent has changed and evolved from its roots in the market place into an elusive concise concept with multiple meanings. According to Parker and Leinhardt, percent can be all the following: (a) a number in that a percent can be written in an equivalent fraction or decimal form; (b) a comparison in the part-whole fraction sense (e.g., if a candidate receives 35% of the votes, this percent is the subset of people who voted for this candidate compared to the total number of votes cast); (c) a ratio comparison where the comparison is between two distinct sets (e.g., there are 400% more boys than girls); (d) a statistic for manageable interpretation (e.g., a state's employment rate of 8.5% is compared to the national average of 10%); and (e) a function when amounts are calculated according to a stated percent (e.g., interest rates, discounts, etc). The link between these many dimensions of percent, according to Parker and Leinhardt, is that of proportionality:

> The common thread woven through all these descriptions is that percent is an alternative language used to describe proportional relationships - a language that is unique, concise and provides a privileged notation system. (p. 444)

Knowing percent: Understanding mathematics requires, amongst other things, proficiency in arithmetical calculation but arithmetical proficiency alone is no guarantee of having mathematical understanding (Leinhardt, 1988). The recent literature reveals that understanding mathematics consists of having knowledge of a concept and process in a variety of forms and that the different forms of knowledge need to be connected to form a schema that is accessible in a variety of application tasks (Putnam, Lampert, & Peterson, 1990). Of pertinence to this paper are the knowledge forms of Resnick (1982), Skemp (1978) and Leinhardt (1986). Resnick categorised knowledge as being either syntactic (the correct performance of mathematical procedures) or semantic (the understanding of the meaning of those procedures), Skemp categorised knowledge as being either instrumental (knowledge of computational procedures) or relational (knowledge of why those procedures work) whilst Leinhardt categorised knowledge as being intuitive ('everyday' real world application knowledge which is normally acquired before formal instruction), concrete (associated with representation by appropriate concrete materials during instruction), computational (knowledge of the algorithmic procedures) or principled/conceptual (knowledge of the principles that constrain or justify the algorithmic procedures and which takes place after instruction is complete). Thus, to the cognitivist, knowing mathematics various internalised representations of related mathematical ideas and connections between the representations (Putnam, Lampert & Peterson, 1990).

Understanding percent, then requires appropriate mental models to accommodate the various notions of percent as well as the procedures solving percent problems. As Parker and Leinhardt (1995, p. 47) stated: "... knowing percent both in school and out means understanding its multiple and often embedded meanings and its relational character".

However, to solve problems, students also need to access the knowledge they have constructed. Prawat (1989) argued that access to knowledge is determined by the learner's organisation and awareness of three factors: knowledge base (concepts, principles, rules, facts and procedures); strategic and metastrategic thinking (general problem solving heuristics and metacognitive processes, such as planning, monitoring, checking, revising); and disposition (habits of mind). In particular, performance on mathematical tasks is influenced by metacognition. Garofalo and Lester (1985) argued that mathematical knowledge is influenced by three metacognitive categories of person knowledge, "one's assessment of one's own capabilities and limitations with respect to mathematics in general, and also with particular topics or tasks" (pp. 167) including such affective variables as motivation, anxiety and perseverance; task knowledge, one's beliefs about the nature of the mathematical tasks; and strategy knowledge, awareness of strategies for guiding problem solving. Thus adequate percent knowledge consists of understanding the meaning of percent in its many dimensions together with knowledge of the principles which legitimise percent calculations, as well as metacognitive knowledge to enhance access to such percent knowledge.

Instructional approaches for teaching percent: The literature provides a variety of instructional methods for developing the concept of percent and for solving percent application problems. A common approach used to develop the concept of percent is to link percents to fractions and decimals (e.g., Brueckner & Grossnickle, 1987). However, the concept percent can also be promoted through linking percent to ratio (e.g., Brown & Kinney, 1973), through studying percent expressions as statements of proportion (e.g., Schmalz, 1977), through exploring the special language of percent, and through the exploration of patterns of simple percent calculations (e.g., Glatzer, 1984). Instructional approaches for solving percent application problems are also varied. In terms of mathematics structure, percent application problems are of three types, described by Ashlock, Johnson, Wilson and Jones (1983, p. 297) in the following manner:

Type I Finding a part or percent of a number (e.g., 25% of 20 is p);

Type IIFinding a part or percent one number is of another (e.g., p% of 15 is 5); and

Type III Finding a number when a certain part or percent of that number is known (e.g., 20% of p is 6).

In their analysis of the literature on percent, Parker and Leinhardt (1995) stated that, by 1960, there were 5 distinct computational procedures for solving percent equations taught in schools. The five procedures can be summarised as follows:

(1) Traditional/cases - students classify the problem and apply a different procedure for each problem type (multiply the number by the percent as a decimal for Type I, divide

the numbers and translate the decimal answer to a percent for Type II, and divide the number by the percent as a decimal for Type III);

(2) Percent formula - "knowns" are substituted in the formula, P = BR (P is percent as a number, B is base number and R is percent as a rate) and the "unknown" is found by algebraic manipulation;

(3) Equation - "knowns" are categorised as factors or product and substituted in the formula, factor x factor = product (algebraic manipulation is used to find the unknown);

(4) Proportion - percent is considered as a common fraction with a denominator of 100 and is equated to a fraction made up from the two other possible numbers (i.e. a/100 = c/d), the problem is classified in terms of the unknown and this is found by algebraic manipulation or cross-multiply method; and

(5) Unitary - 1% of the "known" is calculated and then simple arithmetic computations are performed to calculate the required percent (e.g., 11% of 200 is thought of as the product of 1% of 200 and 11).

The literature offers various teaching approaches to give meaning to the variety of computational procedures. For example, representing percent problems with 10x10 grids (a large square divided into 10 rows of 10 small squares) or on number lines (from 0 to 100) have been suggested as a means for helping students visualise the computational procedures of percent calculations (e.g., Bennett & Nelson, 1994). Mnemonic strategies, which emphasise the key words "of" (meaning multiply) and "is" (meaning divide) have also been suggested to help students interpret percent problems and to order percent calculations (e.g., McGivney & Nitschke, 1988). However, these approaches rarely appear to address the multi-faceted nature of the topic of percent, cover all knowledges (decimal/fraction, ratio, and proportion), and encompass all percent meanings (number, fraction, ratio, proportion, statistics and function). Furthermore, Parker and Leinhardt (1995) reported that results of comparative teaching studies did not conclusively suggest that one approach was superior to another.

Assessment of percent knowledge: Parker and Leinhardt (1995) claimed that percent is a confusing topic in the mathematics curriculum for both students and teachers, and that basically, "percent is hard" (p. 423). Their claim is supported by the findings with respect to percent of the Fourth National Assessment of Educational Performance (NAEP) of Mathematics (Kouba et al., 1988) which provided evidence that students at the 7th and 11th grade levels appear to lack understanding of percent and have difficulty with percent applications (particularly Types II and III problems).

A recent study by Lembke and Reys (1994) looked at Years 5, 7, 9 and 11 students' conceptual and computational percent knowledge, before and after formal percent instruction, and showed a more promising picture of the percent knowledge students may possess. Lembke and Reys interviewed high- and middle-ability students in each of the four year levels and found that: (a) students in Years 5 and 7 (who had not received formal instruction in percent used a variety of intuitive strategies to solve percent problems; (b) older students (Years 9 and 11) utilised a percent formula for calculating percentages, often making careless errors; and (c) common benchmarks (100% is a whole, 50% is half, and 25% is half of a half of something) were used by students of all year levels as an aid to undertaking the calculations and to check the reasonableness of their calculations.

Implications for this paper: In a similar manner to the study of Lembke and Reys (1994), the study on which this paper reports was designed to analyse and categorise percent knowledge and solution strategies accessed by students of different proficiency with respect to percent problem solving, and to draw implications for appropriate instruction in the development of percent concepts and solution strategies. Specifically, the study focused on three year levels (Years 8, 9 and 10) and three categories of proficiency: proficient, able to solve all three types of percent problems; semi-proficient, able to solve type I problems but not able to solve types II and III problems; and non-proficient, not able to solve any type of problem. The following questions were a focus for the study:

(1) What knowledge do proficient, semi-proficient and non-proficient percent problem solvers possess and access?

(2) How do proficient, semi-proficient, and non-proficient percent problem solvers interpret and represent percent problems?

This paper reports on three aspects of this study, namely, proficient, semi-proficient and non-proficient percent problem solvers' knowledge of percent problem types, type of solution strategy, and use of diagrams.

Method

The methodology adopted in the study is qualitative. The research method is that of semi-structured Piagetian clinical interview and protocol analysis (Ginsburg, 1981).

Subjects: The subjects were a purposeful sample of eighteen students from a Year 8, a Year 9 and a Year 10 class from a Brisbane secondary boys school. The ninety students from these three classes were given the three types of percent problems to solve and, from their responses, were categorised as proficient, semi-proficient, or non-proficient. From this, two students per year level were selected at random from each of the performance categories.

Instruments: The instrument was a clinical interview. The tasks focused on students' understanding of percent problems (designed to be within the experience of the students) and the strategies the students used in solving these problems. The first task explored students' global schema of percent by asking about their knowledge of the three structural types of percent problems; the second identified strategies used by the students in solving percent problems by asking them how they solved the three types of problems; the third explored students' use of diagrams in solving percent problems by asking them to solve a problem with a diagram if they did not spontaneously use one.

Procedure: The students were removed from their class and interviewed in a separate room. The interviews lasted 30 minutes and were videotaped. The students had attempted the problems before the interview and the interview focused on recalling the methods they had used in solution. The probing of the students was based on contingent questions. If knowledge was detected that had not been used in problems, the students were questioned as to why it was not used.

Analysis: The interviews were subjected to protocol analysis (Ericcson & Simon, 1984). The grounded theory approach of Strauss and Corbin (1990) was used to determine patterns and commonalities.

Results

The eighteen students' responses to the three tasks are given in detail. The students are denoted as follows (the first number refers to their Year level):

Proficient students	8P1, 8P2, 9P1, 9P2, 10P1, 10P2
Semi-proficient students	8SP1, 8SP2, 9SP1, 9SP2, 10SP1, 10SP2
Non-proficient students	8NP1, 8NP2, 9NP1, 9NP2, 10NP1, 10NP2

Students' responses on number of percent problem types: Four of the six proficient student (8P2, 9P2, 10P1 and 10P2) and two semi-proficient students (9SP2 and 10SP2) identified the three types of problems from Ashlock et al. (1983). One non-proficient student (8NP2) identified two of the problem types, describing them as 'find the percent of a number given - what number is 25% of 60' and 'find what percent of one number another number is another - 15 is what percent of 60'; as did one semi-proficient student (8SP2), describing the problem types as 'finding the percent of a number, determining what percent of this number is this number, profit problems and loss problems'. The remaining 10 students (8P1, 9P1, 8SP1, 9SP1, 10SP1, 8NP1, 9NP1, 9NP2, 10NP1 and 10NP2) did not identify any of the Ashlock et al. problem types, and thought there were four or more types. The non-proficient students were particularly varied in their descriptions of problem types. For example, 'questions on maths tests, percent in the real world, percentages used to sell things, and percentages used for exporting' (8NP1) and 'those ones which you divide and multiply' (10NP2).

Students' solution strategies: The most widely used strategy for finding solutions to percent problems was the percent formula procedure. This was used by three proficient students (9P1, 9P2, 10P2), four semi-proficient students (9SP1, 9SP2, 10SP1, 10SP2) and all the non-proficient students. This strategy was used with a trial-and-error strategy (when the formula was forgotten) by two proficient students (9P1, 9P2) and three semi-proficient students (9SP1, 10SP1, 10SP2). All the non-proficient students did not attempt the problems for which they could not determine a formula.

Two proficient students (8P1, 9P1) used the unitary procedure, two proficient students (8P2, 10P1) used the traditional/cases procedure, while two semi-proficient students (8SP1, 8SP2) used the proportion procedure. Four non-proficient students (9NP1, 9NP2, 10NP1, 10NP2) used a key words strategy (the word "of" indicates multiply and "is" indicates divide).

Proficient students showed strong skills in mental computation and operation relationships (8P1, 9P1, 9P2, 10P1), conversions between percent, common fractions and decimal fractions (8P1, 10P1, 10P2), and benchmarking, approximation and estimation (8P1, 8P2, 9P1, 9P2, 10P1); they also showed some indication of ability to analyse the problems structurally (8P2, 9P1, 9P2, 10P1). Examples of bench marking, approximation and estimation are provided by the following: 8P1 said that the answer to '51 is 85% of what number?' had to be a little larger than 51 because of the relationship of 85% to the whole; 9P1 related '186 is what percent of 240?' to 120 out of 240 being 50%; while, for '28% of 150?', 10P1 said 28% is approximately 1/3, which means that the answer is close to 50. The proficient students used estimation with trial and error and knowledge of structure, for example, when given two numbers and asked to find a percent, 9P2 divided the smaller number by the larger and multiplied by 100, looked at the answer, and reversed what he had done when he thought the answer was unreasonable.

All semi-proficient students used some aspects of benchmarking, approximation and estimation to assist their problem solving but were not as skilled as the proficient students in mental computation. Some semi-proficient students were also skilful with conversions (8SP2, 9SP1).

Non-proficient students did not generally reveal flexible thinking; rather, they tended to follow routinised patterns of activity (e.g., convert the percent to a decimal) even if that strategy was not helping them to solve problems. Some non-proficient students did this before they even read the problem.

Students' responses with respect to use of diagrams: No students spontaneously drew diagrams in solving the percent problems. When asked to use this strategy to help solve the problems, all the proficient students, one semi-proficient student (9SP2) and five non-proficient students (8NP1, 8NP2, 9NP1, 9NP2, 10NP2) were able to draw diagrams that reflected the problem. The remainder could not draw a diagram and were not interested in doing so. Of the students who drew diagrams, four (8P2, 9P1, 8NP2, 10NP2) drew number lines, three (8P1, 10P2, 9NP2) drew pie charts, three (10P1, 9SP2, 9NP1) drew 10x10 grids, one (8NP1) drew rough rectangles, and one (9P2) drew diagrams of rivers and used the analogy of people crossing these rivers with respect to the problems.

Discussion and conclusions

Performance across categories: Proficient students generally knew that there were three types of problems. They could also represent percents in these problems with a variety of effective diagrams, although they were reluctant to draw them and tended not to use them to solve the problems. With regard to their solution strategies, they found it frustrating to discuss their procedures for solution and their preferred response was "I just do it!". They did not rely on the percent formula approach as much as less proficient students; they tended to use a variety of approaches. When an approach was not available (e.g., they forgot the formula), they tried another approach. In their solutions, they constantly estimated and benchmarked, manipulated numbers until the answers makes sense, converted readily between percents, common fractions and decimal fractions, and had a good understanding of the relative size of numbers in terms of relationships in the problem (in this, they tended to have the multiple meanings of percent as described by Parker & Leinhardt, 1995). They appeared to have good mental calculation skills and to understand the effect of operations (e.g., they reversed operations). Importantly, they also appeared to be able to analyse problems in terms of their meanings and be able to predict the operation to be used and the size of the answer relative to the numbers they had been given (e.g., when given problem '51 is 85% of what?', they could see that 51 was approximately 3/4 of the answer).

Except for one student (9SP2), semi-proficient students had no idea of the number of percent problem types and could not represent percent situations with diagrams. With respect to problem solutions, they were reliant on the percent formula approach although they were happy to use trial and error if they forgot the formula. Like the proficient students, they used benchmarking, approximation and computational estimation, but usually as a checking mechanism at the end of the solution rather than as an aid to analyse the problem and predict the size of the solution at the beginning of the problem. They were able to realise when an answer did not make sense, but were unable to construct alternative strategies to correct their mistakes or overcome difficulties.

Non-proficient students thought there were many types of percent problems, usually seeing surface features as constituting difference (e.g., percent to sell things and percent to import are different problems). Surprisingly, they were able to draw (when asked) appropriate diagrams to represent the problem but were unable to use them for solution. With respect to solution strategies, they tended to try to solve problems by using the formula and key words approaches rather than examine the question as a whole; they looked for "of" for multiplication and "is" for division (following a syntactic approach, Resnick, 1982). They had little idea if an answer was sensible or if they had used the formula correctly. Unlike the proficient students, their repertoire of strategies was limited so that, if they forgot the formula (their main strategy), they were unable to access another strategy and consequently were unable to solve the problem.

Expectations of performance: Proficient students were able to access knowledge that enabled them to solve all three types of percent problems. Therefore it is likely that the proficient students had principled-conceptual and relational knowledge (Leinhardt, 1988; Skemp, 1978) and strategic and metastrategic thinking (Prawat, 1989), including self belief and strategy knowledge (Garofalo & Lester, 1985). Thus, it seems reasonable to expect proficient students to use solution strategies that efficiently translate their schema-based understanding of percent to solutions, that is, either the traditional cases or the proportion procedure. On the other hand, less proficient students were unable to access knowledge useful to percent problem solving. Therefore, it seems likely that less-proficient students had procedural and instrumental knowledge (Leinhardt, 1988; Skemp, 1978) of percent and lacked strategic and metastrategic thinking (Garofalo & Lester, 1985; Prawat, 1989). Thus, it seems reasonable to expect that less-proficient students would use rote procedures inflexibly in attempting to solve percent problems.

The students' responses in this study were as expected for non-proficient and semi-proficient students. These students were inflexible and formula oriented. The semi-proficient students showed more use of estimation and trial and error than the non-proficient students. The non-proficient students tended to focus on key words and to discontinue solution attempts if they could not determine an appropriate formula.

However, students' responses were not as expected for proficient students in that they did not strongly reflect a schematic understanding. Proficient students did show some indication of identifying problems by solution structure and they knew the problem types. They also used strategies and metastrategies and were confident in their solutions (as expected from the findings of Garofalo & lester, 1985). However, instead of a schema-based interpretation of problems leading to a classification approach to solution (e.g., cases, proportion), they tended to use a flexible mixture of benchmarking, approximation and estimation, and number and operation sense, along with a variety of strategies and some use of the trial and error strategy (i.e., what could be called a 'first principles' approach to solution).

Implications for teaching

The first implication is based upon the reasons for the success of the proficient students (and the lack of success of the other students), a success which lay in the proficient students' repertoire of strategies, flexibility with respect to strategy access, and number and operation skills. Therefore, the skills of benchmarking, approximation and estimation, conversions between percent, decimals fractions and common fractions, and number and operation sense (including mental computation) should be the focus of instruction for all students, along with the trial and error strategy.

The second implication is based upon the unexpected strategy use of the proficient students. Of the 90 students who were tested, only a very few could be categorised and selected as proficient. These few students did not translate their knowledge into efficient solution procedures (although they did translate their knowledge into effective procedures). Therefore, there appears to be a need for students to be able to recognise problem type and to translate this directly to solution procedure. Instruction in proportion using a number-line diagram appears to offer the best opportunity for this because it does not rest totally on recognition of categories (as the cases procedure does).

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